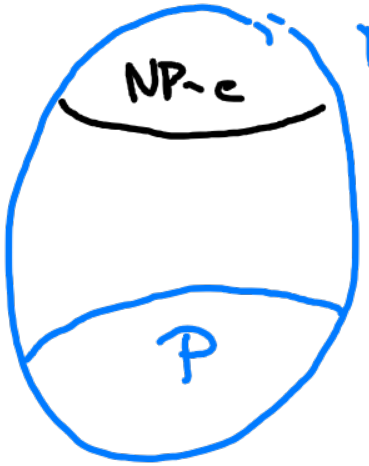
Announcements

- Hw5 interview question changed!!! on Monday
  note: proof & run time always required!
- Hw4 grades released with lessons learned on Ed
- Hw6 released after class
- Mid-term course eval and separate: survey on sections (email from me with the link) (so far 102 responses)
- Study abroad program opportunity focusing in CS

Please read rules posted above hw questions

Reminder: P, NP, NP-complete and SAT



NP = yes/no decision problem, where can be verified in poly time with appropriate help (hint)

NP-complete = hardest problems in NP

P = yes/no decision problems, solvable in poly time

First hardest problem in NP (Cook & Levin '71)

SAT
satisfiability

Input formula $\varnothing = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_1 \vee \bar{x}_3)$

example yes
$x_1 = T$
$x_2 = F$

n variables $x_1 \dots x_n$

m clauses, each or of some literals x_i or $\bar{x}_i$ (= not x_i)

connected with AND

Question: Is there a way to set variables true or false to make $\varnothing$ true

Proving things NP-complete, and $X \leq_p Y$

Problem Y at least as hard as X ($X \leq_p Y$)

if a poly time algorithm for Y can be used to solve X in poly time

Y is NP-complete if for $\forall\ X \in NP$ $X \leq_p Y$

Recepie to prove problem NP-complete

1. problem in NP
2. select an NP-complete problem X
3. Prove your problem $\geq_p X$

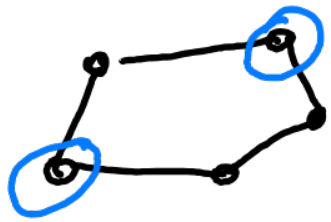
Recall: any poly time algorithm for any one NP complete problem

$1M prize

Claim: Independent Set is NP-complete

Indep set input $G=(V,E)$ & k Question does G have $I \subseteq V$

I indep (no two connected by edge) $|I| \geq k$



$k=2$

1: Indep Set in NP seen on Monday

2. SAT

3. need $SAT \leq_p$ Indep Set

need: way to solve SAT using Indep Set alg.

need given $\varphi = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_1 \vee x_3)$

$\Rightarrow$ turn into graph G & k

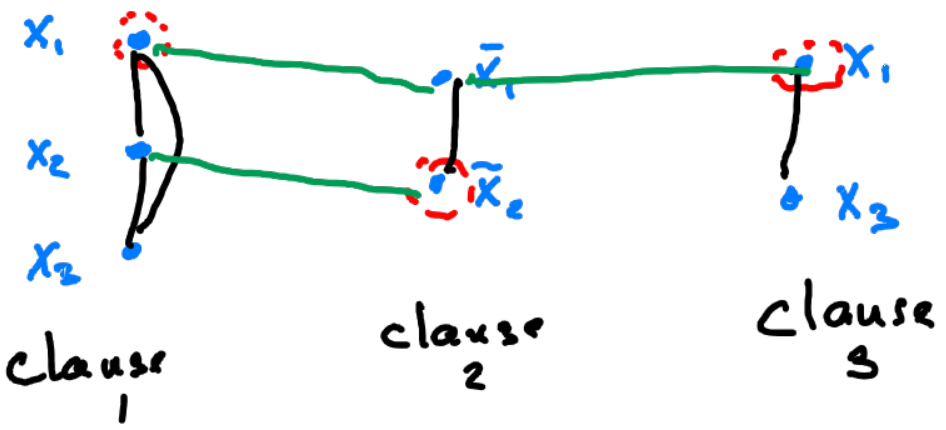
Idea: IS selects nodes in G

in φ we select: what? ~ true literals

need at least one in each clause

SAT $\leq_p$ Independent Set construction

Construction: nodes = literals in each clause, k = # clauses



edges:
- connect nodes in same clause
- connect each copy of x_i & $\bar{x}_i$

nodes = all occurences of all literals separately

k = # clauses

edges:
- connect literals in same clause
- connect variable x & $\bar{x}$ all variables

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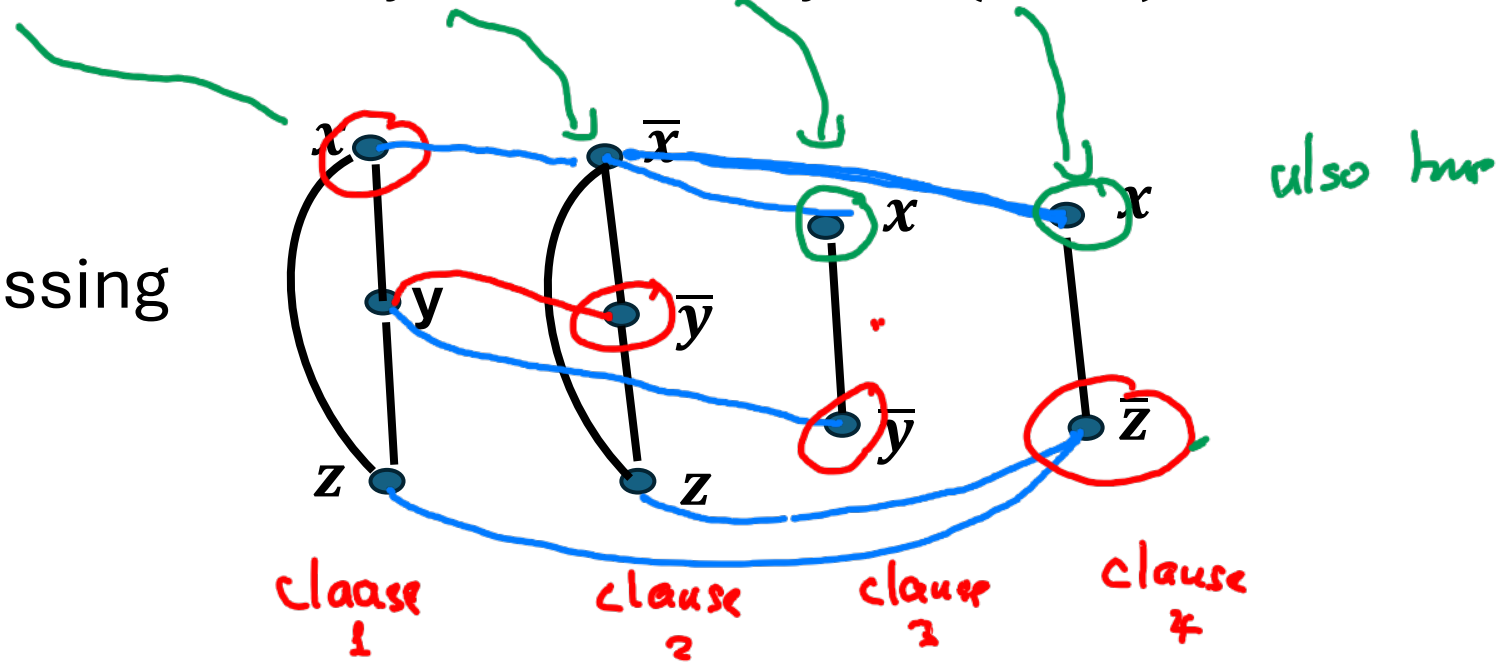
Is this the correct graph for Independence set correspond to the SAT formula $\Phi = (x \vee y \vee z) \wedge (\bar{x} \vee \bar{y} \vee z) \wedge (x \vee \bar{y}) \wedge (x \vee \bar{z})$

A: correct

B: almost, one edge missing

C: more wrong

D: I don't know



Indep set : $x=T$, $y=F$ & $z=F$

Independent Set is NP-complete (proof)

Claim: ϕ formula satisfiable (Yes) if & only if indep set answer Yes

only if 1. G has indep set of size $k = m$ ($\Rightarrow$? ϕ satisfiable?)

clauses are fully connected
$\Rightarrow$ indep set of size m must have a node in each clause

set literal true if selected

OK assignment
edges x & $\bar{x}$ $\Rightarrow$ valid truth assignment ✓

if 2. ϕ is satisfiable ($\Rightarrow$ G has indep set?)

$\phi = (x \vee y \vee z) \wedge (\bar{x} \vee \bar{y}) \wedge \cdots$

pick satisfying assignment

there is at least one literal true in each clause. $\Rightarrow$ corresponding nodes are indep in G

Plans for next week, 3-SAT, 2-SAT and many other problems

in P

We know NP-complete problems so far

- SAT & Indep Set
- Vertex Cover (Monday $VC \geq_p IS$)

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also true 3-SAT NP-complete (all clause have 3 items)

Note 2-SAT is in P

Problem 3 on hw will be easier after Monday.